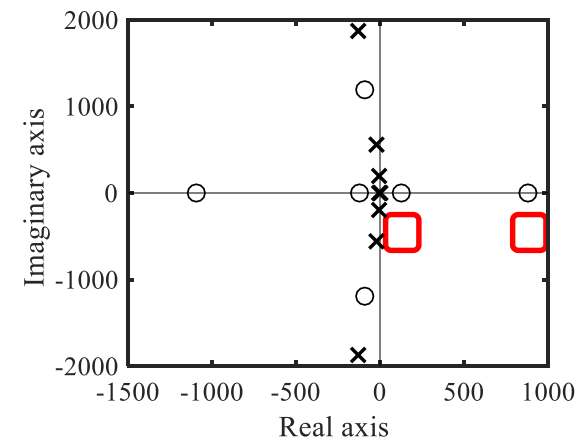
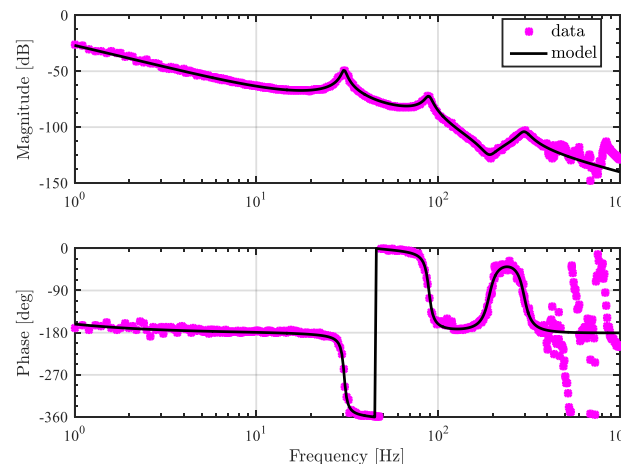
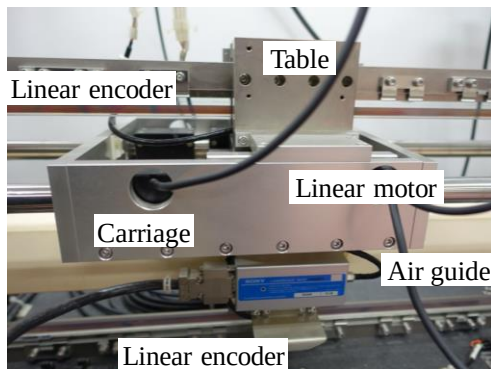


連続時間不安定零点を持つ精密位置決めステージの軌道追従制御法
—時間軸反転とマルチレートフィードフォワードによる安定逆系の設計—
Tracking control method for high-precision stage
with continuous time unstable zeros:
Stable inversion by time axis reversal and multirate feedforward

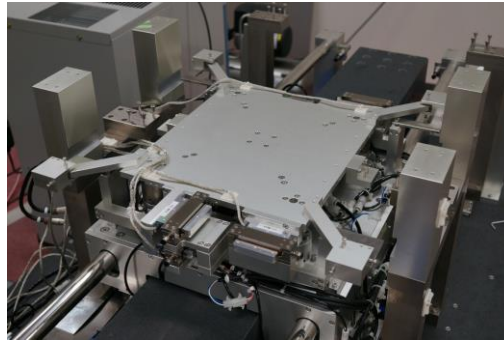
Wataru Ohnishi, Thomas Beauduin, and Hiroshi Fujimoto
The University of Tokyo



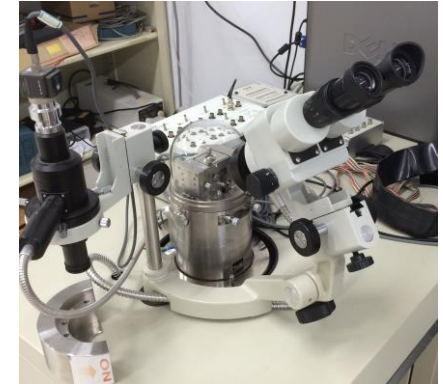
1.1 Introduction – examples of the plant with unstable zeros –



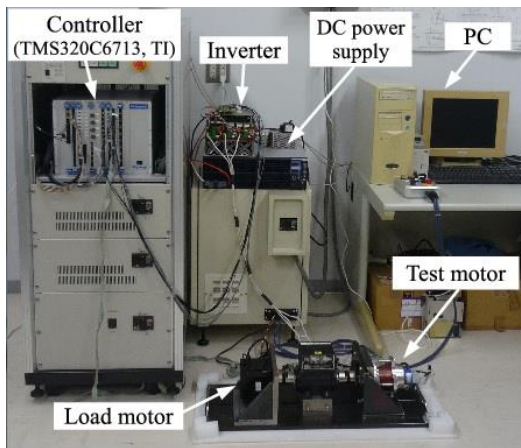
Hard Disk Drive



High-precision stage



Atomic Force Microscope



Motor and converter



Robot



Airplane

<http://hflab.k.u-tokyo.ac.jp/>

1.2 Zeros in discrete time domain

$$P_c(s) = \frac{-(s-140)(s+100)}{s(s+2000)(s+2)(s^2+20s+40000)}$$

$$P_s[z] = \frac{K(z+3.547)(z-1.014)(z-0.9900)(z+0.2543)}{(z-1)(z-0.9998)(z-0.8187)(z^2-1.998z+0.998)}$$

Discretization by
zero-order hold
($T_u = 100\mu s$)

- **Intrinsic zeros (真性零点)**
have counterpart in continuous time zeros
Generated from sensor and actuator collocation

- **Discretization zeros (離散化零点)**
are generated by discretization
Approximated as Euler-Frobenius polynomial

Relative order	Discretization zeros
2	-1
3	$-2 - \sqrt{3}, (-2 - \sqrt{3})^{-1}$ ($\approx -3.7, -0.26$)

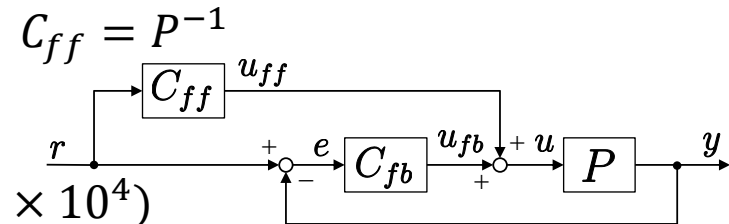
Åström, K., Hagander, P. and Sternby, J.: Zeros of sampled systems, *Automatica*, Vol. 20, No. 1, pp. 31–38, 1984.
T. Hagiwara, T. Yuasa, and M. Araki, "Stability of the limiting zeros of sampled-data systems with zero-and first-order holds," *Int. J. Control*, vol. 58, no. 6, pp. 1325–1346, 1993.

1.3 Introduction – problem of unstable zeros –

- Its inversion system has unstable poles

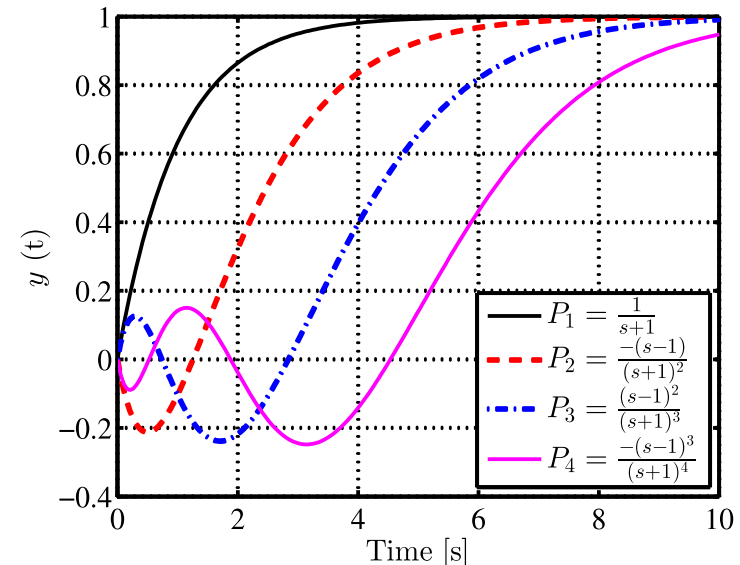
$$P_c^{-1}(s) = \frac{s(s + 84.65)(s^2 + 22.21s + 1.151 \times 10^4)}{-0.1034(s + 231.6)(s - 93.65)}$$

$$P_s^{-1}(z) = \frac{(z - 1)(z - 0.9916)(z^2 - 1.998z + 0.9978)}{-5.175 \times 10^{-10}(z + 1.001)(z - 1.009)(z - 0.9771)}$$



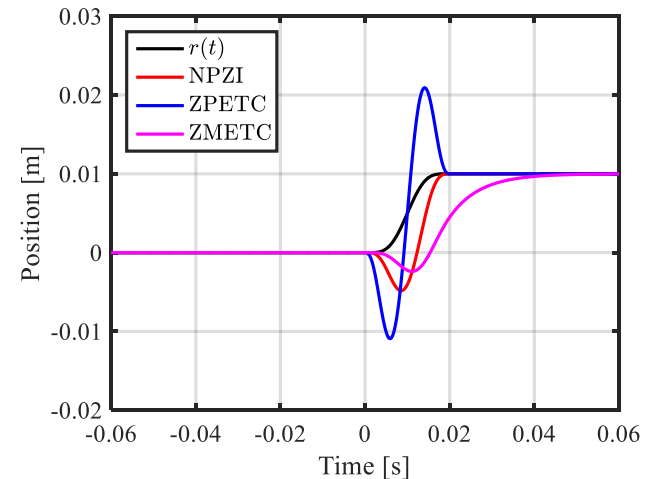
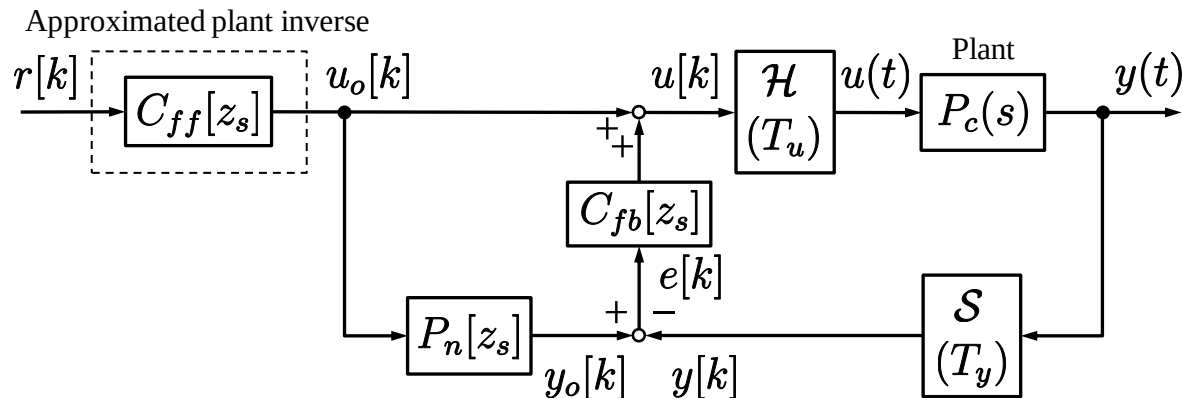
- Undershoot
Tradeoffs between settling time t_s and
% Undershoot M_u
(b_0 denotes unstable zeros)

$$M_u > \frac{1}{b_0 t_s}$$



Hoagg, J. and Bernstein, D.: Nonminimum-phase zeros – much to do about nothing - classical control - revisited
part II, IEEE Control Systems, Vol. 27, No. 3, pp. 45–57 (2007)

2. Approximated inverse based single rate feedforward



- Plant approximated inverse is designed in **discrete time**
Discretization zeros and intrinsic zeros are dealt in same time

$$P_n[z_s] = \frac{B[z_s]}{A[z_s]} = \frac{B_s[z_s]B_u[z_s]}{A[z_s]} \quad \longrightarrow \quad C_{ff}[z_s] = \tilde{P}_n^{-1}[z_s] = \frac{z^{-q}A[z_s]}{B_s[z_s]B_u^*[z_s]}$$

Stable part: $B_s[z_s]$

Unstable part: $B_u[z_s]$

$B_u[z_s]$ is approximated as $B_u^*[z_s]$

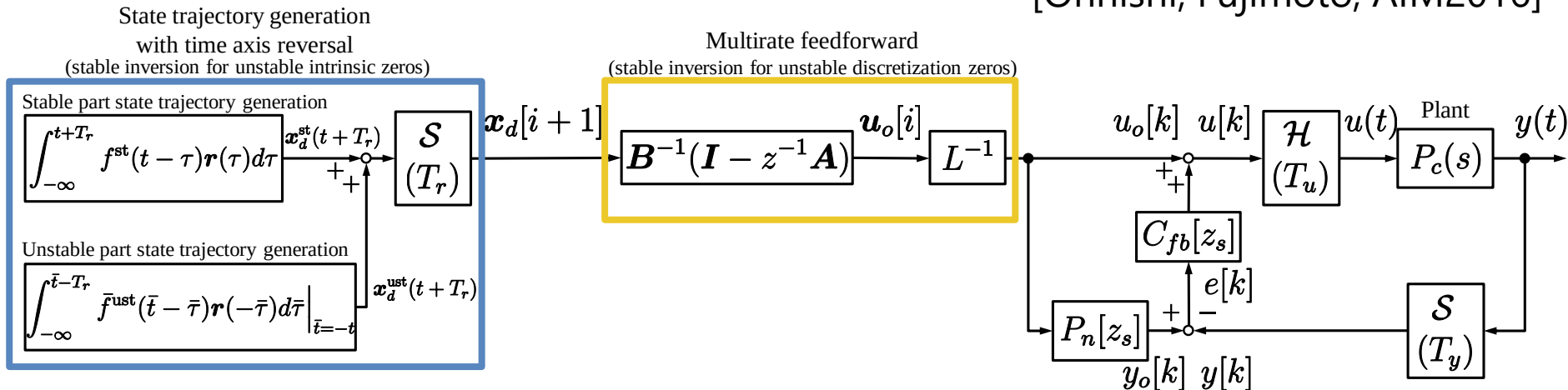
and $C_{ff}[z_s] = \tilde{P}_n^{-1}[z_s]$ is designed

1. nonminimum-phase zeros ignore (NPZI) method [Butterworth, et al. 2012]
2. zero-phase-error tracking controller (ZPETC) method [Tomizuka, 1987]
3. zero-magnitude-error tracking controller (ZMETC) method [Wen & Potsaid, 2004]

Trade off between zero-phase-error and zero-magnitude-error characteristics

3. Preactuation Perfect Tracking Control (PPTC) method

[Ohnishi, Fujimoto, AIM2016]



1. State trajectory generation

Stable inversion for continuous unstable zeros by time axis reversal

$$P_c(s) = \frac{B(s)}{A(s)} \mathbf{x}_d(t) = \frac{1}{\textcircled{B(s)}} \mathbf{r}(t)$$

$\mathbf{x}_d(t)$ diverges when $B(s)$ is unstable

→ time axis reversal

2. Multirate feedforward

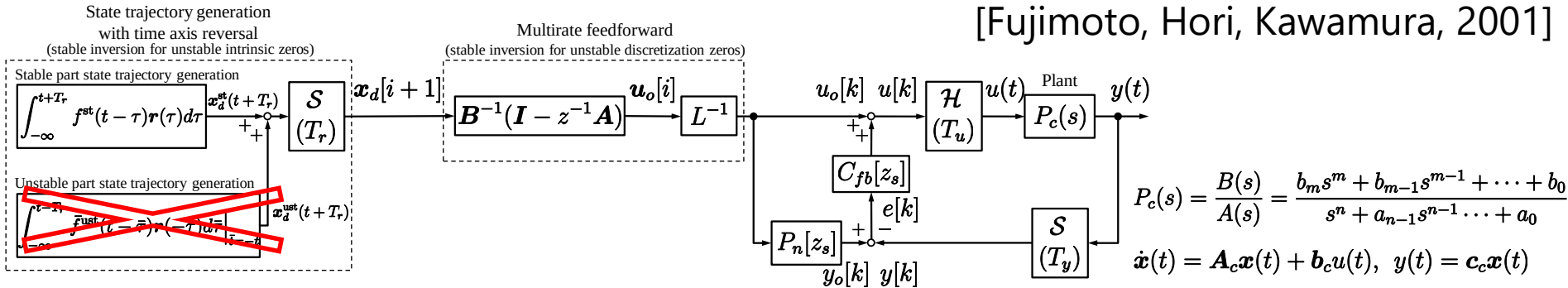
Stable inversion for unstable discretization zeros [Fujimoto, Hori, Kawamura, 2001]

$$\mathbf{u}_o[i] = \mathbf{B}^{-1}(\mathbf{I} - z^{-1}\mathbf{A}) \mathbf{x}_d[i + 1]$$

Stable trajectory generation is a key to achieve PTC for NMP system

3.1 Conventional PTC method –State trajectory generation–

[Fujimoto, Hori, Kawamura, 2001]



$$r(t) = y_d(t) = \mathbf{c}_c \mathbf{x}_d(t)$$

$$= \underbrace{\begin{bmatrix} b_0 & b_1 & \dots & b_m & 0 & \dots & 0 \end{bmatrix}}_{\text{Coeffs of numerator}} \underbrace{\begin{bmatrix} x_{1d} \\ s x_{1d} \\ \vdots \\ s^{n-1} x_{1d} \end{bmatrix}}_{\text{Derivative relationship (Control Canonical Form)}}$$

$$x_{1d}(t) = \frac{1}{(b_0 + b_1 s + \dots + b_m s^m)} r(t)$$

$$= \frac{1}{B(s)} r(t)$$

$$f(t) = \bar{\mathcal{L}}^{-1} \left[\frac{1}{B(s)} \right] = \bar{\mathcal{L}}^{-1} \left[\frac{1}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0} \right]$$

Diverge for plant with continuous unstable zeros

→ Approximated in conv study [Fukushima, Fujimoto, et al., 2006]

$$\mathbf{x}_d(t) = \int_0^t f(t - \tau) \mathbf{r}(\tau) d\tau$$

$$\mathbf{r}(t) = \underbrace{\begin{bmatrix} 1 & \frac{d}{dt} & \dots & \frac{d^{n-1}}{dt^{n-1}} \end{bmatrix}^T}_{\text{Position, velocity, acceleration...}} \mathbf{r}(t)$$

3.2 Preactuation PTC method –State trajectory generation–

1. Stable & unstable decomposition $P_c(s) = \frac{B(s)}{A(s)} = \frac{b_ms^m + b_{m-1}s^{m-1} + \dots + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_0}$ [Ohnishi, Fujimoto, 2016]

$$B^{-1}(s) = F^{\text{st}}(s) + F^{\text{ust}}(s) \quad f^{\text{st}}(t) = \bar{\mathcal{L}}^{-1} [F^{\text{st}}(s)], \quad \bar{f}^{\text{ust}}(t) = \bar{\mathcal{L}}^{-1} \left[\underbrace{(-1)^l F^{\text{ust}}(-s)}_{\text{Stabilized by imaginary axis reversal}} \right]$$

**Stabilized by
imaginary axis reversal**

2. State trajectory generation for stable part

$$\begin{aligned} \mathbf{x}_d^{\text{st}}(t) &= \begin{bmatrix} x_{1d}^{\text{st}}(t) & x_{2d}^{\text{st}}(t) & \dots & x_{nd}^{\text{st}}(t) \end{bmatrix}^T \\ &= \int_{-\infty}^t f^{\text{st}}(t - \tau) \mathbf{r}(\tau) d\tau \end{aligned}$$

3. State trajectory generation for unstable part

$$\begin{aligned} \mathbf{x}_d^{\text{ust}}(t) &= \begin{bmatrix} x_{1d}^{\text{ust}}(t) & x_{2d}^{\text{ust}}(t) & \dots & x_{nd}^{\text{ust}}(t) \end{bmatrix}^T \\ &= \int_{-\infty}^{\bar{t}} \bar{f}^{\text{ust}}(\bar{t} - \bar{\tau}) \mathbf{r}(-\bar{\tau}) d\bar{\tau} \Big|_{\bar{t}=-t} \end{aligned}$$

Convolution between
time axis reversed reference
and
imaginary axis reversed unstable zeros,
and then time axis reversed again

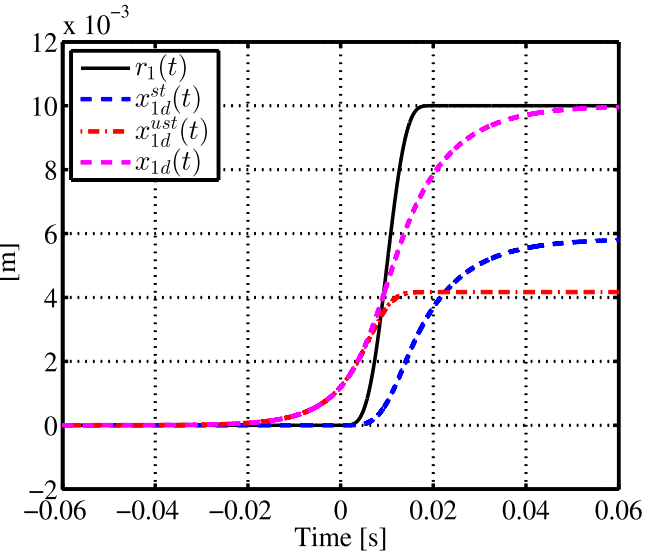
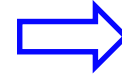
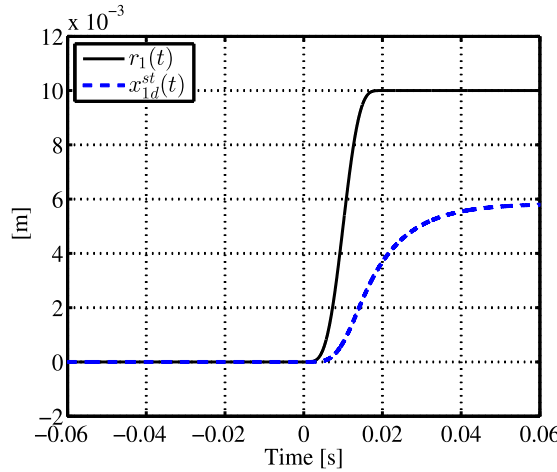
[Sogo, 2006]

4. Overall state trajectory $\mathbf{x}_d(t) = \mathbf{x}_d^{\text{st}}(t) + \mathbf{x}_d^{\text{ust}}(t)$

3.2 Preactuation PTC based on multirate FF

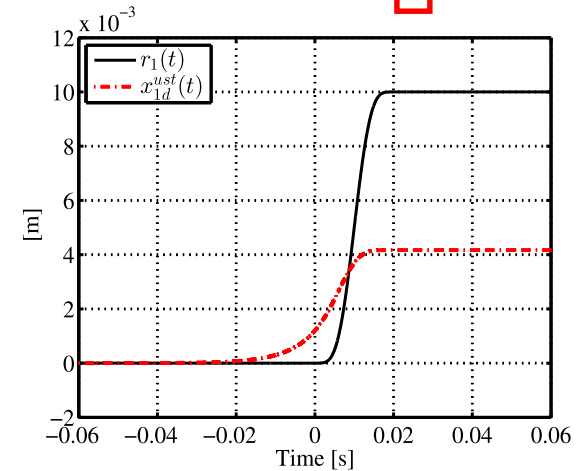
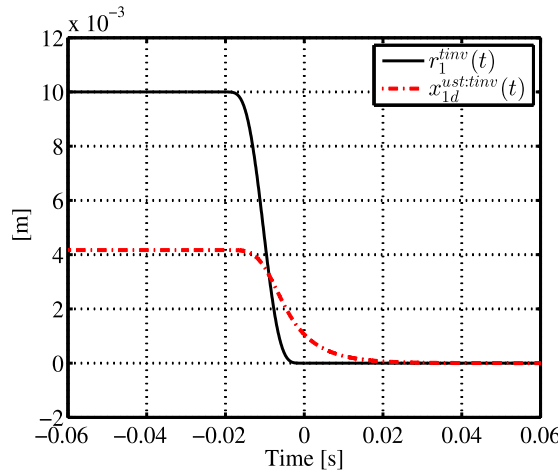
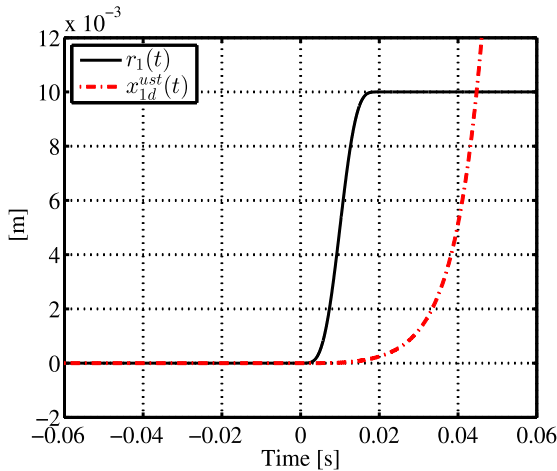
Example

$$P_c(s) = \frac{(s - 140)(s + 100)}{D(s)}$$



$$x_{1d}^{st} = \frac{a^{st}}{s + 100} r_1$$

$$x_{1d}(t) = x_{1d}^{st}(t) + x_{1d}^{ust}(t)$$

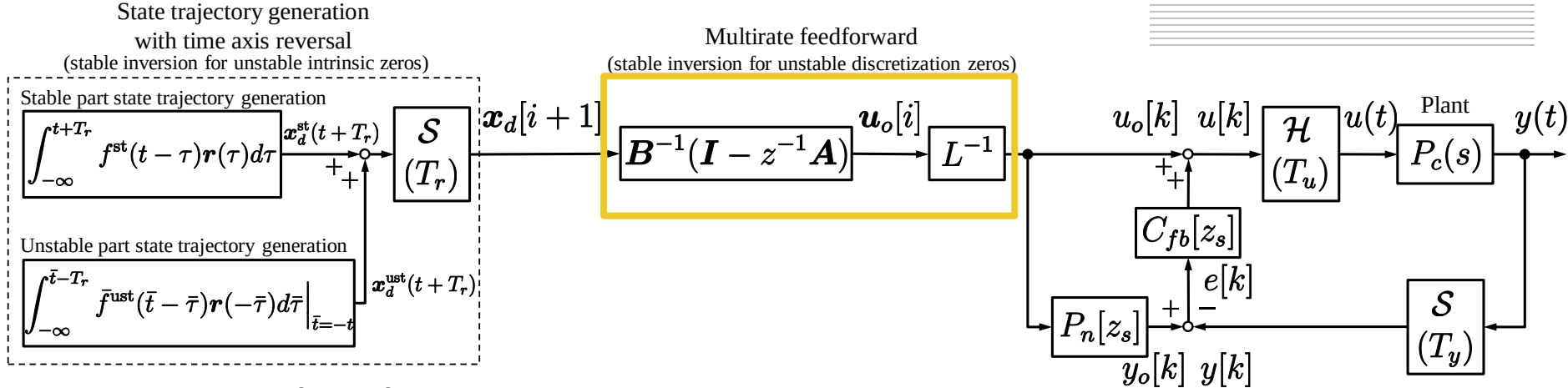


$$x_{1d}^{ust} = \frac{a^{ust}}{s - 140} r_1$$

$$x_{1d}^{ust:inv} = \frac{a^{ust}}{s + 140} r_1^{inv}$$

$$x_{1d}^{ust}(t) = x_{1d}^{ust:inv}(-t)$$

3.3 Preactuation PTC based on multirate FF



- Multirate feedforward

[Fujimoto, Hori, Kawamura, 2001]

Single rate system $\mathbf{x}[k + 1] = \mathbf{A}_s \mathbf{x}[k] + \mathbf{b}_s u[k], \quad y[k] = \mathbf{c}_s \mathbf{x}[k]$

Multirate system $\mathbf{x}[i + 1] = \mathbf{A} \mathbf{x}[i] + \mathbf{B} \mathbf{u}[i], \quad y[i] = \mathbf{c} \mathbf{x}[i]$

$$\mathbf{A} = \mathbf{A}_s^n, \quad \mathbf{B} = [\mathbf{A}_s^{n-1} \mathbf{b}_s \quad \mathbf{A}_s^{n-2} \mathbf{b}_s \quad \cdots \quad \mathbf{A}_s \mathbf{b}_s \quad \mathbf{b}_s]$$

$$\mathbf{c} = \mathbf{c}_c, \quad \mathbf{x}[i] = \mathbf{x}(iT_r)$$

$\mathbf{B}$ matrix become non-singular

$$\mathbf{u}_o[i] = \mathbf{B}^{-1}(\mathbf{I} - z^{-1} \mathbf{A}) \mathbf{x}_d[i + 1]$$

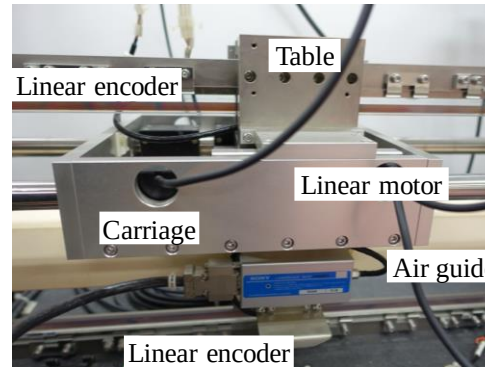
$$z = e^{sT_r}$$

$$T_r = nT_u$$

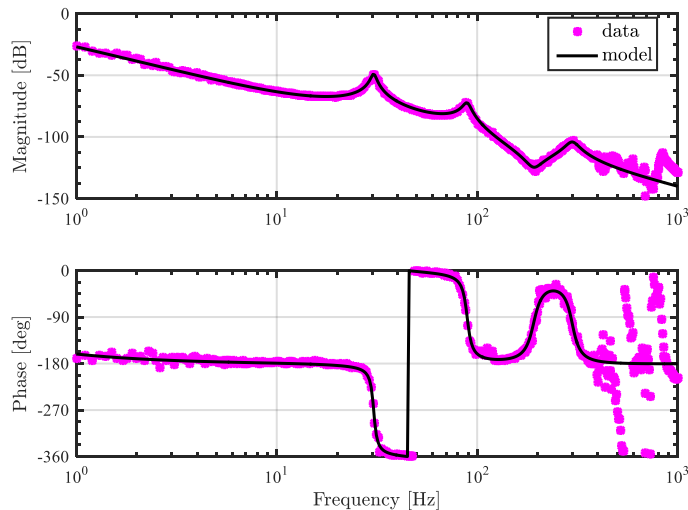
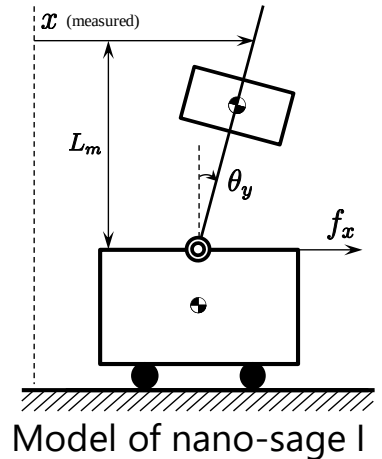
Stable inversion for discretization zeros!

3.4 Experimental setup

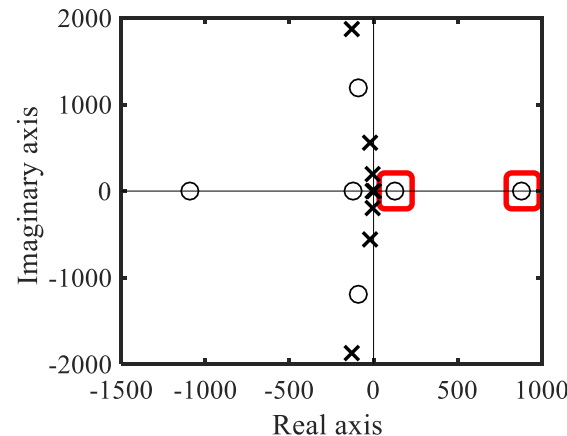
- High-precision stage
Guide: air guide
Motor: Linear motor
Encoder: 1nm resolution $\times 2$
- Frequency response
From measured current to measured position



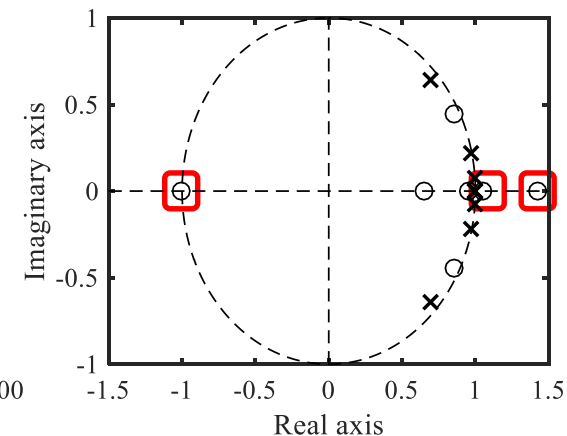
Nano-sage I



Frequency response of plant
8th order model



Unstable cont. zeros $\times 2$

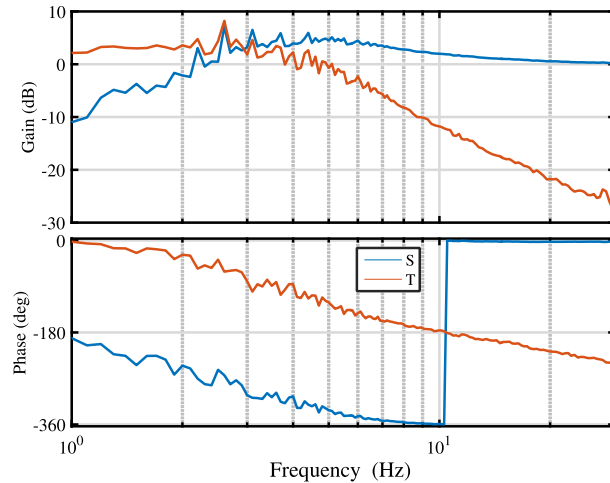


Unstable intrinsic. zeros $\times 2$
Unstable discretization zeros $\times 1$

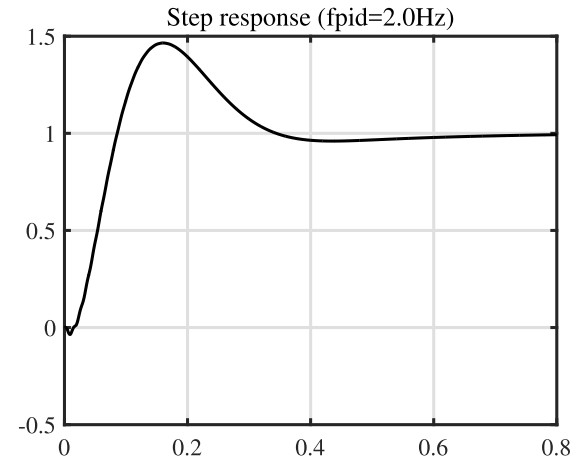
$$P_c(s) = \frac{3.50(s + 1090)(s - 878)(s - 126)(s + 120)(s^2 + 186s + 1.45 \times 10^6)}{s(s + 2.10)(s^2 + 10.9s + 3.67 \times 10^4)(s^2 + 45.4s + 3.14 \times 10^5)(s^2 + 262s + 3.51 \times 10^6)}$$

$$P_d[z_s] = \frac{2.70 \times 10^{-7}(z_s + 1.01)(z_s - 0.953)(z_s - 1.05)(z_s - 1.42)(z_s - 0.647)(z_s^2 - 1.71z_s + 0.929)}{(z_s - 1)(z_s - 0.999)(z_s^2 - 1.99z_s + 0.996)(z_s^2 - 1.93z_s + 0.982)(z_s^2 - 1.39z_s + 0.900)4}$$

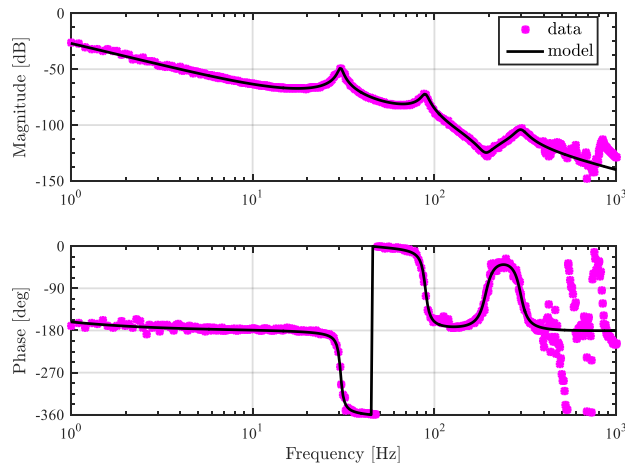
3.4 Simulation and Experiment –FB controller–



Closed loop performance



Step response (simulation)



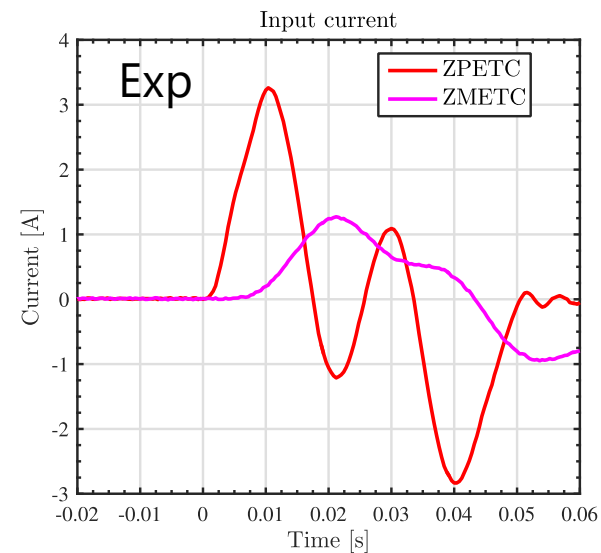
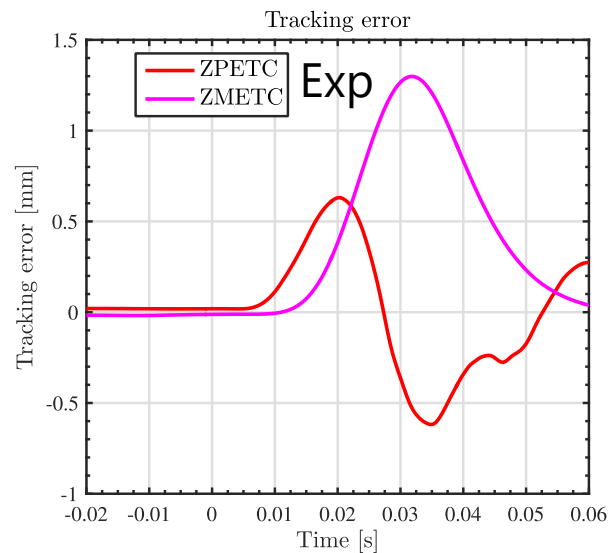
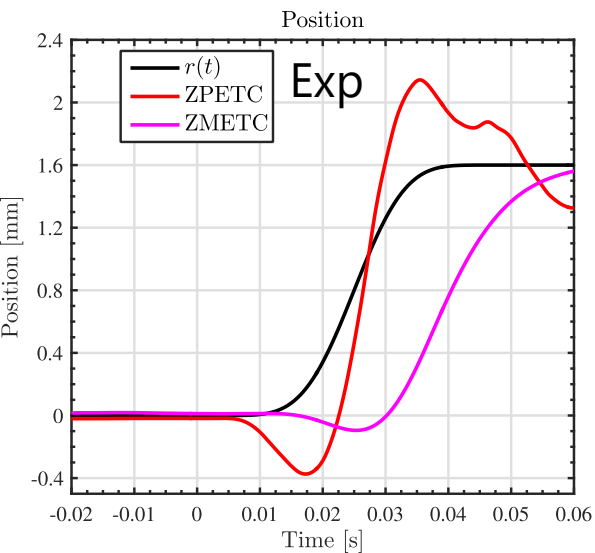
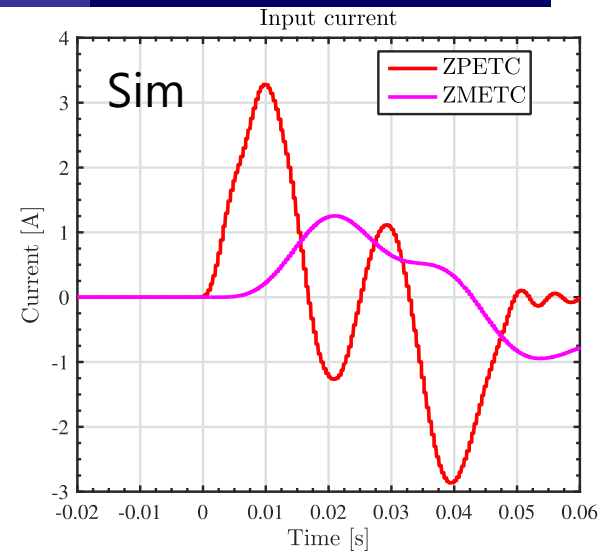
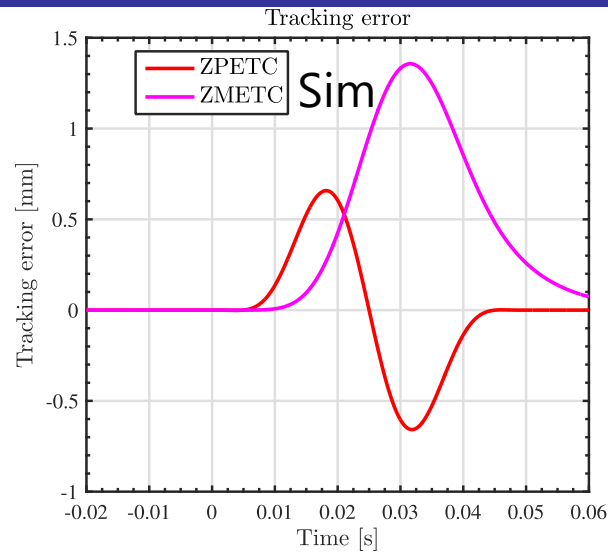
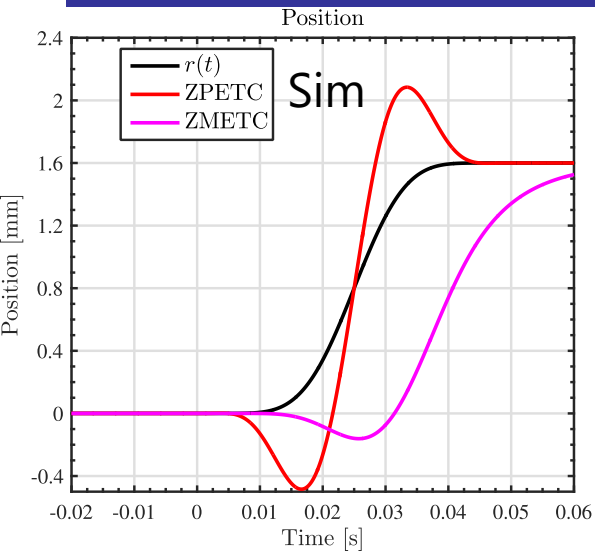
Plant

FB controller

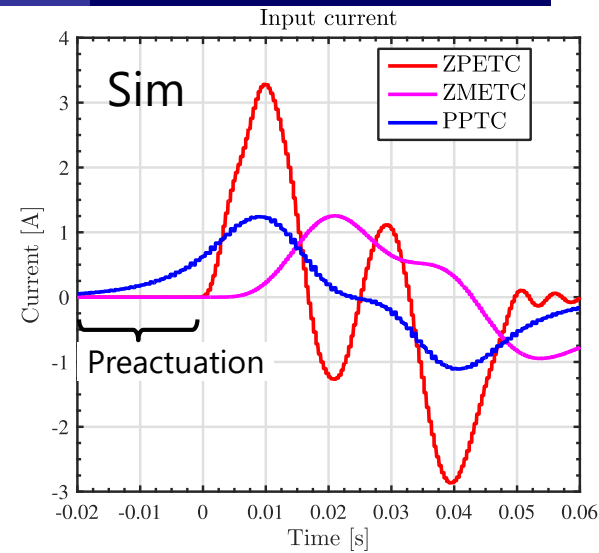
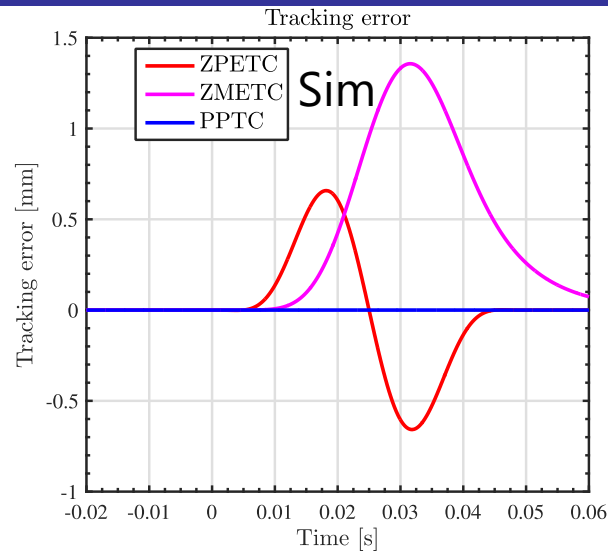
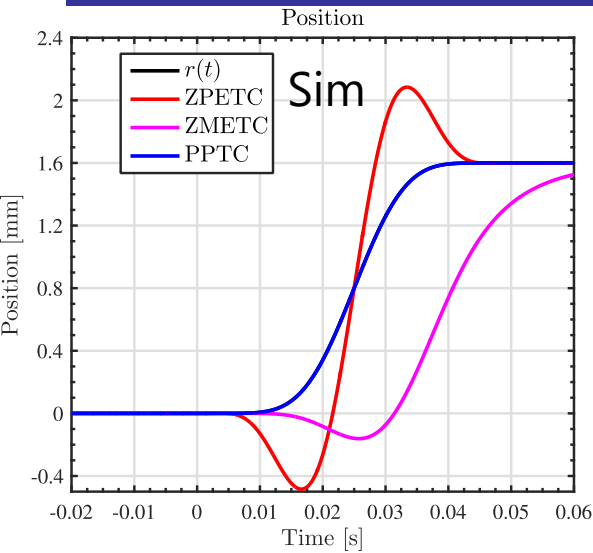
- PID by pole placement design (2 Hz)
- Shaping filter for 30Hz NMP zero

FB Bandwidth: 2.3 Hz

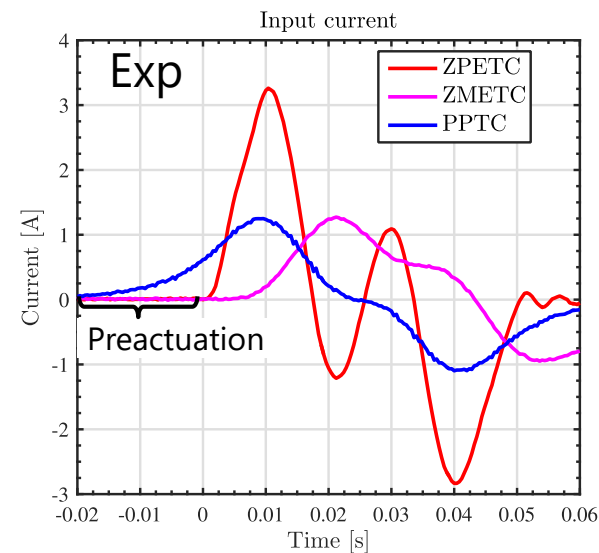
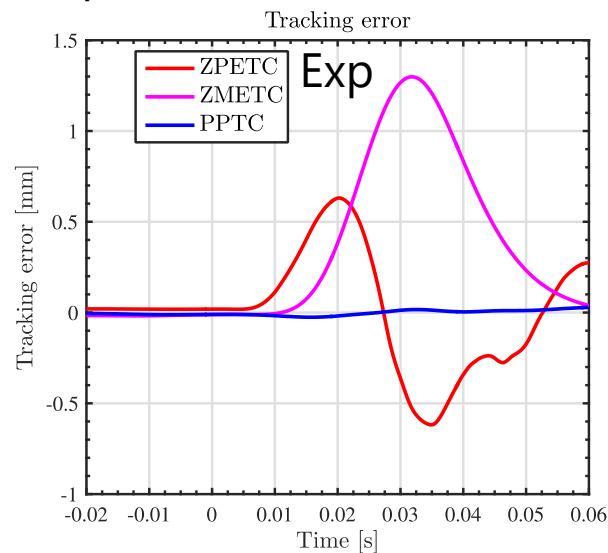
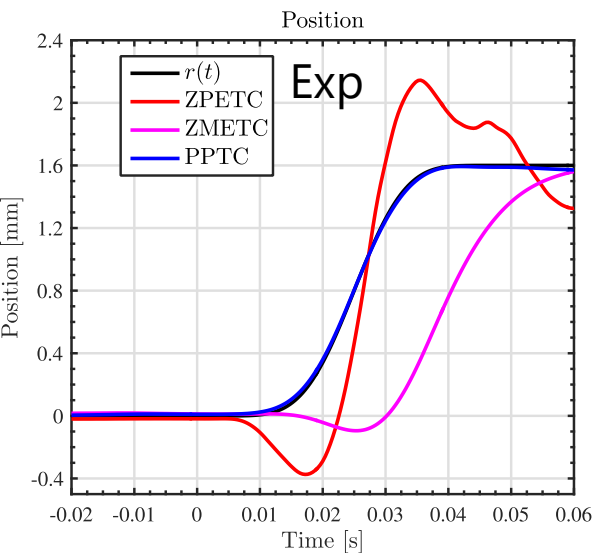
3.4 Simulation and Experiment



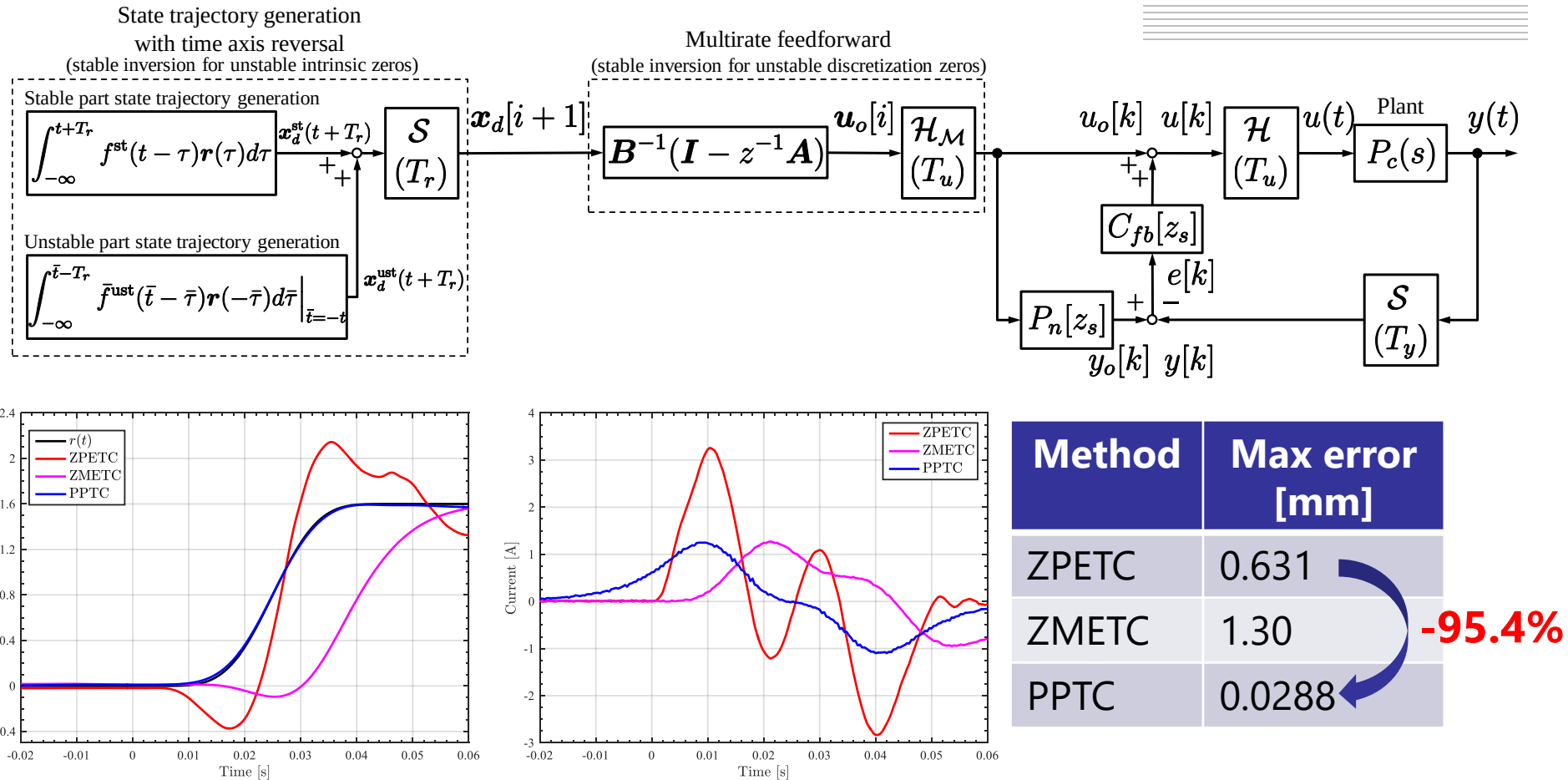
3.4 Simulation and Experiment



PPTC: Preactuation perfect tracking control
Input current from -0.4 [s]



4. Conclusion



- Preactuation Perfect Tracking Control achieves perfect tracking for plant with continuous unstable zeros by infinite time preactuation